Determination of invariant deformation parameters from GPS permanent stations using stochastic spatial interpolation

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The data: time series of GPS network coordinates \( x_i(t) \)

Temporal smoothing interpolation

Models for temporal coordinate variations \( x_i(t,a_i), a_i = \text{model parameters} \)

\[ x_i(t,a_i) = x_0 + \gamma_i \] (linear model \( a_i = x_0, \gamma_i \))

Trend removal of displacements by reference system redefinition

Spatial Interpolation (finite elements), stochastic prediction (collocation)

Statistical analysis of residuals \( x_i(t_i) - x_i(t_i,a_i) \)

Coordinates \( x(P,t,a_i) \) at any area point \( P \) at any epoch \( t \)

Full covariance matrix \( C_a \) of model parameters \( a_i \) for all stations \( P_i \)

Gradient matrix

\[ \mathbf{F} = \frac{\partial x(t')}{\partial x(t)} \]

Covariance propagation

Covariance matrices \( C_{\text{vec}(P_i)} \) and cross covariances \( C_{\text{vec}(P_i)\text{vec}(Q)} \) for any area points \( P_i, Q \)

New rigorous equations (no infinitesimal approximation)

Invariant deformation parameters (for earth surface or its projection on ellipsoid)

\( \theta \) - direction of principal axes
\( \lambda_1, \lambda_2 \) - principal elongations
\( \Delta \) - dilatation
\( \gamma \) - maximum shear strain
\( \psi \) - direction of \( \gamma \)

Covariances and cross-covariances of \( \theta, \lambda_1, \lambda_2, \Delta, \gamma, \psi \) - for any area points

The simulation: a displacement field simulated on a regular grid.

Grid extension: 100 km x 100 km, cell size 2 km x 2 km.

Displacement observations extracted without noise for a network of 9 points.

Statistics of the simulated displacements (2500 grid cells, values in cm)

<table>
<thead>
<tr>
<th>Component</th>
<th>Mean</th>
<th>RMS</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>X component</td>
<td>0.0</td>
<td>2.5</td>
<td>-4.9</td>
<td>7.4</td>
</tr>
<tr>
<td>Y component</td>
<td>0.0</td>
<td>2.6</td>
<td>-5.1</td>
<td>5.1</td>
</tr>
</tbody>
</table>

The prediction

From the displacement observations on the 9 network points, by collocation, on the whole grid:

- displacements, gradient matrices and deformation parameters
- covariances and cross-covariances of the deformation parameters.

In the above figure, the predicted displacements on the grid:

- Green: positions of the measurement points.
- Red: displacements, depicted with a magnification factor of \( 5 \times 10^4 \).
- Light gray: predicted values outside the network boundary.

Below, the statistics of the differences between the simulated and the predicted displacements.

The values (in cm) are computed only inside the network boundary.

The values (in cm)

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>X component</td>
<td>0.1</td>
<td>0.4</td>
<td>-1.1</td>
<td>1.5</td>
</tr>
<tr>
<td>Y component</td>
<td>0.3</td>
<td>0.4</td>
<td>-0.5</td>
<td>1.2</td>
</tr>
</tbody>
</table>

New rigorous equations (no infinitesimal approximation)

Covariances and cross-covariances of \( \theta, \lambda_1, \lambda_2, \Delta, \gamma, \psi \) - for any area points

Note: the shears RMSs are similar to the dilatations RMSs.

Light gray: predicted values outside the network boundary.

Green: positions of the measurement points.

Black: unit circle.

Red: maximum eigenvalue.

Gray: predicted values outside the network boundary.

Depicted values: \( \lambda = \lambda_{max} - 1) \), \( \lambda = 5 \times 10^3 \)

Note: correlations between max and min eigenvalues are very close to zero.