The problem:
For a homogeneous constant-mean random field \( u(P) \)
\[ E\{u(P)\} = \mu, E\{[u(P) - \mu][u(Q) - \mu]\} = C(P, Q) = C(0) \]
Given noisy observations \( y_i = L_i(u) + v_i, i = 1, \ldots, n, \)
\( L_i = \) linear functionals
Predict the value of another linear functional \( L_p(u) \).
Usual case: \( L_i(u) = u_j = u(P_j) \) \& \( L_p(u) = u(P) \)

Observed: \( y = u + v \)
Known:
\( \mu = E\{u\} = \mu_s, s = [1 \ 1 \ \ldots \ 1]^T, C = E\{(u - \mu)(u - \mu)^T\}, \)
\( E\{v\} = 0, \ C_v = E\{v v^T\}, \ C_{uv} = E\{(u - \mu)v^T\} = 0 \)
\( E\{u(P)\} = \mu, \ c_p = E\{(u - \mu)(u(P) - \mu)\} \)
Required:
Linear prediction \( \hat{u}(P) = \hat{u}(P) \) of \( u(P) \)
with minimum Mean Square Prediction Error
\[ m_p^2 = E\{e^2\} = E\{[\hat{u}(P) - u(P)]^2\} = \min \]
\[ m_p^2 = \min m_p^2 \]

Reduction to a finite-dimensional model: **Linear Random Effects Model**

### In terms of the covariance function

<table>
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<th>Unbiased</th>
<th>Bias</th>
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<tr>
<td>( C(P, Q) = E{[u(P) - \mu][u(Q) - \mu]} )</td>
<td>( C_0 = C(P, P) = C(0) )</td>
<td>( \beta = E{u(P)} - E{u} )</td>
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**Unbiased**

\[ \hat{u}(P) = d^T y + k \]

\( \beta = 0 \)

\[ \hat{m}_p^2 = C_0 - c_p^T (C + c_p)^{-1} c_p \]

**Biased**

\[ \hat{u}(P) = \mu + c_p^T (C + c_p)^{-1} (y - \mu) \]

\[ \alpha_s = \frac{s^T (C + c_p)^{-1} y}{s^T (C + c_p)^{-1} s} \]

\[ \beta = \frac{\mu^T - c_p^T (C + c_p)^{-1} \mu}{1 + \mu^T (C + c_p)^{-1} \mu} \]

\[ \hat{m}_p^2 = C_0 - c_p^T (C + c_p)^{-1} c_p + \mu^T \frac{[1 - c_p^T (C + c_p)^{-1} s]^2}{1 + \mu^T (C + c_p)^{-1} s} \]

### In terms of the variogram

\( \gamma(P, Q) = \frac{1}{2} E\{[u(P) - u(Q)]^2\} \)

\( \Gamma_u = \gamma(P, P) \)

\( (\gamma_P, \gamma) = \gamma(P, P) \)

**Unbiased**

\[ \hat{u}(P) = \alpha_s + \gamma_P (\Gamma - C_s)^{-1} (y - \alpha_s s) \]

\[ \alpha_s = \frac{s^T (\Gamma - C_s)^{-1} y}{s^T (\Gamma - C_s)^{-1} s} \]

\[ \beta = 0 \]

\[ \hat{m}_p^2 = H + \gamma_P (\Gamma - C_s)^{-1} \gamma_P = \frac{[\gamma_P^2 (\Gamma - C_s)^{-1} s]^2}{s^T (\Gamma - C_s)^{-1} s} \]

**Biased**

\[ \hat{u}(P) = \alpha_B + \gamma_P (\Gamma - C_s)^{-1} (y - \alpha_B s) \]

\[ \alpha_B = \frac{H s^T (\Gamma - C_s)^{-1} y}{H s^T (\Gamma - C_s)^{-1} s - 1} \]

\[ \beta = \frac{\mu^T - \gamma_P (\Gamma - C_s)^{-1} \mu}{H s^T (\Gamma - C_s)^{-1} s - 1} \]

\[ \hat{m}_p^2 = \gamma_P (\Gamma - C_s)^{-1} \gamma_P \frac{H (s^T (\Gamma - C_s)^{-1} y - 1)^2}{H s^T (\Gamma - C_s)^{-1} s - 1} \]

Unknown mean \( \mu \) not present !

Unknown mean \( \mu \) present only through estimable \( H = C_0 + \mu^2 \)

Biased Kriging is feasible !