Computation of Earth Rotation Parameters Consistent with the IERS Earth Rotation Representation

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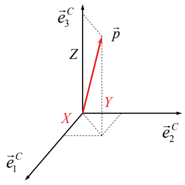
IERS Terrestrial (T) to Celestial (C) Coordinate Transformation:

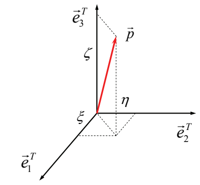
$$\mathbf{R}=\underbrace{\mathbf{Q}_0(X,Y)\mathbf{R}_3\big(s(X,Y)\big)}_{\text{Precession-Nutation}}\underbrace{\mathbf{R}_3\big(-\theta(\tau)\big)}_{\text{Diurnal Rotation}}\underbrace{\mathbf{R}_3\big(-s'(x_p,y_p)\big)\mathbf{R}_2(x_p)\mathbf{R}_1(y_p)}_{\text{Polar Motion}}$$

Geocentric bases: $\vec{\mathbf{e}}^C=[\vec{e}_1^C\ \vec{e}_2^C\ \vec{e}_3^C]$ $\quad\vec{\mathbf{e}}^T=[\vec{e}_1^T\ \vec{e}_2^T\ \vec{e}_3^T]$

Position vector: $\vec{x}=\vec{\mathbf{e}}^C\mathbf{x}_C=\vec{\mathbf{e}}^T\mathbf{x}_T$

Base & coordinate transformation: $\vec{\mathbf{e}}^C=\vec{\mathbf{e}}^T\mathbf{R}^T$ $\quad\mathbf{x}_C=\mathbf{R}\mathbf{x}_T$





Involves CIP (Celestial Intermediate Pole): $\vec{p}=\vec{\mathbf{e}}^C\mathbf{p}_C=\vec{\mathbf{e}}^T\mathbf{p}_T$, $\quad|\vec{p}|=1$, $\quad\mathbf{p}_C=\begin{bmatrix}X\\Y\\Z\end{bmatrix}$, $\quad\mathbf{p}_T=\begin{bmatrix}\xi\\\eta\\\zeta\end{bmatrix}\approx\begin{bmatrix}x_p\\-y_p\\\zeta\end{bmatrix}$

with 5 Transformation Parameters: $X,\ Y,\ x_p,\ y_p,\ \tau\equiv\mathrm{UT1}-\mathrm{UTC}$ $\quad$ 2 more than 3 required for defining the orthogonal matrix $\mathbf{R}$ $\rightarrow$ **Inconsistency !**

The CIP is determined from theoretical solutions to the earth rotation problem and removal of frequencies beyond the observational resolution

The (mathematically) Compatible Rotation Vector: $\vec{\mathbf{e}}^T=\vec{\mathbf{e}}^C\mathbf{R}\ \Rightarrow\ \frac{d}{dt}\vec{\mathbf{e}}^T=\vec{\mathbf{e}}^C\frac{d\mathbf{R}}{dt}=\vec{\mathbf{e}}^T\mathbf{R}^T\frac{d\mathbf{R}}{dt}\equiv[\boldsymbol{\omega}_T\times]\vec{\mathbf{e}}^T$, $\quad\vec{\omega}=\vec{\mathbf{e}}^T\boldsymbol{\omega}_T=\vec{\mathbf{e}}^C\boldsymbol{\omega}_C$, $\quad\boldsymbol{\omega}_C=\mathbf{R}\boldsymbol{\omega}_T$

$$[\boldsymbol{\omega}_C\times]=\frac{d\mathbf{R}}{dt}\mathbf{R}^T\qquad[\boldsymbol{\omega}_T\times]=\mathbf{R}^T\frac{d\mathbf{R}}{dt}\qquad\omega\equiv|\vec{\omega}|=\sqrt{\boldsymbol{\omega}_T^T\boldsymbol{\omega}_T}=\sqrt{\boldsymbol{\omega}_C^T\boldsymbol{\omega}_C}$$

The (mathematically) Compatible Celestial Pole (CCP): $\vec{n}\equiv\frac{\vec{\omega}}{\omega}=\vec{\mathbf{e}}^C\mathbf{n}_C=\vec{\mathbf{e}}^T\mathbf{n}_T$ $\quad\mathbf{n}_C=\frac{1}{\omega}\boldsymbol{\omega}_C\equiv\begin{bmatrix}\tilde{X}\\\tilde{Y}\\\tilde{Z}\end{bmatrix}$ $\quad\mathbf{n}_T=\frac{1}{\omega}\boldsymbol{\omega}_T=\begin{bmatrix}\tilde{\xi}\\\tilde{\eta}\\\tilde{\zeta}\end{bmatrix}\approx\begin{bmatrix}\tilde{x}_p\\-\tilde{y}_p\\\tilde{\zeta}\end{bmatrix}$

$$\mathbf{R}=\mathbf{Q}_0(\tilde{X},\tilde{Y})\mathbf{R}_3\big(s(\tilde{X},\tilde{Y})\big)\mathbf{R}_3\big(-\theta(\tilde{\tau})\big)\mathbf{R}_3\big(-s'(\tilde{x}_p,\tilde{y}_p)\big)\mathbf{R}_2(\tilde{x}_p)\mathbf{R}_1(\tilde{y}_p)$$

The CCP is determined from observations only, it is mathematically compatible with the rotation matrix R and it is independent from any earth rotation theory

Computation of the CCP = Computation of "Observed" Precession-Nutation, Polar Motion and Length of the Day: (Dermanis, 2005, Journées des Systèmes de Référence Spatio-Temporels,Warsaw)

$$\begin{bmatrix}\dot{\tilde{X}}\\\dot{\tilde{Y}}\end{bmatrix}=\frac{\dot\theta}{\omega}\begin{bmatrix}X\\Y\end{bmatrix}+\frac{1}{\omega}\begin{bmatrix}1-aX^2&-aXY\\-aXY&1-aY^2\end{bmatrix}\left\{\begin{bmatrix}b_Q^1\\b_Q^2\end{bmatrix}-\begin{bmatrix}\cos\psi&-\sin\psi\\\sin\psi&\cos\psi\end{bmatrix}\begin{bmatrix}b_W^1\\b_W^2\end{bmatrix}\right\}$$

$$\begin{bmatrix}\tilde{\xi}\\\tilde{\eta}\end{bmatrix}=\frac{\dot\theta}{\omega}\begin{bmatrix}\xi\\\eta\end{bmatrix}+\frac{1}{\omega}\begin{bmatrix}1-a'\xi^2&-a'\xi\eta\\-a'\xi\eta&1-a'\eta^2\end{bmatrix}\left\{-\begin{bmatrix}b_W^1\\b_W^2\end{bmatrix}+\begin{bmatrix}\cos\psi&\sin\psi\\-\sin\psi&\cos\psi\end{bmatrix}\begin{bmatrix}b_Q^1\\b_Q^2\end{bmatrix}\right\}$$

$$\psi=\theta-s+s'\qquad a=\frac{1}{1+\sqrt{1-X^2-Y^2}}\qquad a'=\frac{1}{1+\sqrt{1-\xi^2-\eta^2}}$$

$$\omega^2=\dot\theta^2+(b_Q^1)^2+(b_Q^2)^2+(b_W^1)^2+(b_W^2)^2+2\sin\psi(b_Q^1b_W^2-b_Q^2b_W^1)-2\cos\psi(b_Q^1b_W^1+b_Q^2b_W^2)$$

$$\begin{bmatrix}b_Q^1\\b_Q^2\end{bmatrix}=\frac{a^2}{2a-1}\begin{bmatrix}X(Y\dot{X}-X\dot{Y})-\frac{aY}{1-a}(X\dot{X}+Y\dot{Y})\\Y(Y\dot{X}-X\dot{Y})+\frac{aX}{1-a}(X\dot{X}+Y\dot{Y})\end{bmatrix}\qquad\begin{bmatrix}b_W^1\\b_W^2\end{bmatrix}=\frac{a'^2}{2a'-1}\begin{bmatrix}\xi(\eta\dot\xi-\xi\dot\eta)-\frac{a'\eta(\xi\dot\xi+\eta\dot\eta)}{1-a'}\\\eta(\eta\dot\xi-\xi\dot\eta)+\frac{a'\xi(\xi\dot\xi+\eta\dot\eta)}{1-a'}\end{bmatrix}$$

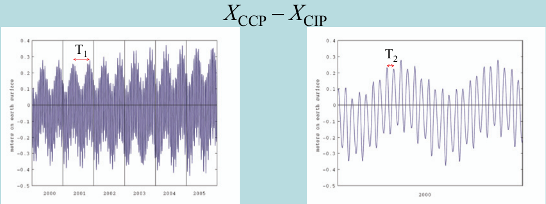
$$\mathbf{R}=\mathbf{Q}_0(\tilde{X},\tilde{Y})\mathbf{R}_3\big(s(\tilde{X},\tilde{Y})\big)\mathbf{R}_3\big(-\theta(\tilde{\tau})\big)\mathbf{R}_3\big(-s'(\tilde{x}_p,\tilde{y}_p)\big)\mathbf{R}_2(\tilde{x}_p)\mathbf{R}_1(\tilde{y}_p)\ \xrightarrow{\text{solve for}}\ \theta(\tilde{\tau})$$

$$\theta(\tau)=A+B\cdot\mathrm{UT1}=A+B(\mathrm{UTC}+\tau)\ \Rightarrow\ \tilde{\tau}=\widetilde{\mathrm{UT1}}-\mathrm{UTC}=\frac{\theta(\tilde{\tau})-A}{B}-\mathrm{UTC}$$

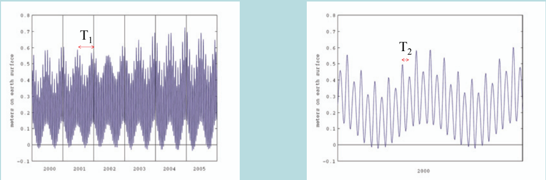
Comparison of Observed Earth Rotation (CCP = Compatible Celestial Pole) with Theoretical Earth Rotation (CIP = Celestial Intermediate Pole)

Precession-Nutation

$X_{\mathrm{CCP}}-X_{\mathrm{CIP}}$



$Y_{\mathrm{CCP}}-Y_{\mathrm{CIP}}$

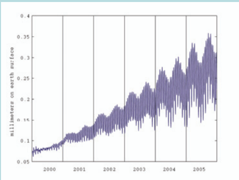


Two periodic terms with periods

T_1 = 190.5 days ($\approx$ semiannual)

T_2 = 13.6 days ($\approx$ half lunar month)

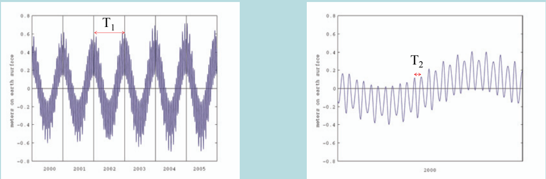
$s_{\mathrm{CCP}}-s_{\mathrm{CIP}}$



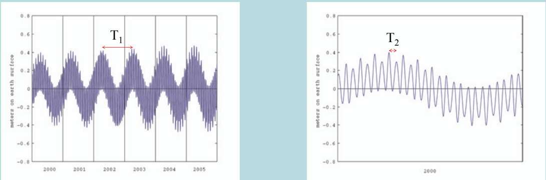
negligible differences

Polar Motion

$(x_{\mathrm{P}})_{\mathrm{CCP}}-(x_{\mathrm{P}})_{\mathrm{CIP}}$



$(y_{\mathrm{P}})_{\mathrm{CCP}}-(y_{\mathrm{P}})_{\mathrm{CIP}}$

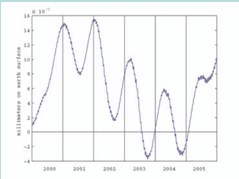


Two periodic terms with periods

T_1 = 357.5 days ($\approx$ annual)

T_2 = 14.2 days ($\approx$ half lunar month)

$s'_{\mathrm{CCP}}-s'_{\mathrm{CIP}}$



negligible differences

Length of the Day

$\tau=\mathrm{UT1}-\mathrm{UTC}$

$\tau_{\mathrm{CCP}}=\tau_{\mathrm{CIP}}$!

Identical !