# On the consistent definition of EOPs in relation to the observed ITRF-ICRF transformation

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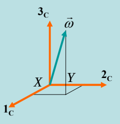
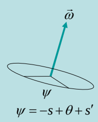
## EARTH ROTATION COMPONENTS

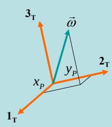
Precession-Nutation $\mathbf{Q}(X,Y)\mathbf{R}_3(s)$

Diurnal Rotation $\mathbf{R}_3(-\theta)$

Polar motion $\mathbf{R}_3(-s')\mathbf{R}_2(x_P)\mathbf{R}_1(y_P)$




celestial reference system $1_C, 2_C, 3_C$

terrestrial reference system $1_T, 2_T, 3_T$

IERS earth rotation representation:

$$\mathbf{R} = \delta\mathbf{Q}(\delta X, \delta Y)\mathbf{Q}_0(X,Y)\mathbf{R}_3(-\psi)\mathbf{R}_2(x_P)\mathbf{R}_1(y_P)$$

Separation by NRO conditions: $\psi = -s(X,Y) + \theta + s'(x_P, y_P)$

Theory provides: $\mathbf{R}_0 = \mathbf{Q}_0(X,Y)\mathbf{R}_3(-\psi_0)$

Observations provide modifications:

$$\mathbf{R} = \delta\mathbf{Q}(\delta X, \delta Y)\mathbf{Q}_0(X,Y)\mathbf{R}_3(-\psi_0)\mathbf{R}_3(-\delta\psi)\mathbf{R}_2(x_P)\mathbf{R}_1(y_P)$$

via infinitesimal rotations $\delta\mathbf{Q}(\delta X, \delta Y)$ $\quad \mathbf{R}_3(-\delta\psi)$ $\quad \delta\mathbf{P} = \mathbf{R}_2(x_P)\mathbf{R}_1(y_P)$

## IMPLICATIONS OF INFINITESIMAL ROTATION PROPERTIES ON EARTH ROTATION SEPARATION

### INFINITESIMAL ROTATIONS AND THEIR PROPERTIES

General infinitesimal rotation:

$$\delta\mathbf{R}(\boldsymbol{\theta}) = \mathbf{R}_3(\theta_3)\mathbf{R}_2(\theta_2)\mathbf{R}_1(\theta_1) \approx \mathbf{I} - [\boldsymbol{\theta}\times] = \begin{bmatrix} 1 & \theta_3 & -\theta_2 \\ -\theta_3 & 1 & \theta_1 \\ \theta_2 & -\theta_1 & 1 \end{bmatrix}$$

Properties of infinitesimal rotations:

(a) Commutation $\quad \delta\mathbf{R}(\boldsymbol{\alpha})\delta\mathbf{R}(\boldsymbol{\beta}) \approx \delta\mathbf{R}(\boldsymbol{\beta})\delta\mathbf{R}(\boldsymbol{\alpha})$

(b) Equivalent "left" & "right" modification of a rotation matrix

$$\delta\mathbf{R}(\boldsymbol{\theta}_L)\mathbf{R} \approx \mathbf{R}\,\delta\mathbf{R}(\boldsymbol{\theta}_R) \qquad \boldsymbol{\theta}_R = \mathbf{R}^T\boldsymbol{\theta}_L \qquad \boldsymbol{\theta}_L = \mathbf{R}\boldsymbol{\theta}_R$$

### Implications of infinitesimal rotation properties:

$$\mathbf{R} = \delta\mathbf{Q}(\delta X, \delta Y)\mathbf{Q}_0(X,Y)\mathbf{R}_3(-\psi_0)\mathbf{R}_3(-\delta\psi)\mathbf{R}_2(x_P)\mathbf{R}_1(y_P)$$

$\mathbf{q} = [\delta Y \quad -\delta X \quad 0]^T$

$\mathbf{p} = [y_P \quad x_P \quad 0]^T$

$$\mathbf{R} = \delta\mathbf{R}(\mathbf{q})\mathbf{Q}_0(X,Y)\mathbf{R}_3(-\psi_0 - \delta\psi)\delta\mathbf{R}(\mathbf{p})$$

$\mathbf{q}' = \mathbf{Q}_0^T\mathbf{q} \equiv [\delta Y' \quad -\delta X' \quad \delta Z']$

$$\mathbf{R} = \mathbf{Q}_0(X,Y)\delta\mathbf{R}(\mathbf{q}')\mathbf{R}_3(-\psi_0 - \delta\psi)\delta\mathbf{R}(\mathbf{p})$$

Left corrections to precession-nutation replaced by equivalent ones from the right

### Miss-interpretation of polar motion

$$\mathbf{R} = \mathbf{Q}_0(X,Y)\delta\mathbf{R}(\mathbf{q}')\mathbf{R}_3(-\psi_0 - \delta\psi)\delta\mathbf{R}(\mathbf{p})$$

$\mathbf{p}' = \mathbf{R}_3(-\psi_0)\mathbf{p} \equiv [y'_P \quad x'_P \quad z'_P]^T$

$$\mathbf{R} = \mathbf{Q}_0(X,Y)\delta\mathbf{R}(\mathbf{q}')\delta\mathbf{R}(\mathbf{p}')\mathbf{R}_3(-\psi_0 - \delta\psi)$$

$\mathbf{q}' = \mathbf{Q}_0^T\mathbf{q} \equiv [\delta Y' \quad -\delta X' \quad \delta Z']$

$$\mathbf{R} = \mathbf{Q}_0(X,Y)\mathbf{R}_3(\delta Z')\mathbf{R}_2(-\delta X')\mathbf{R}_1(\delta Y')\mathbf{R}_3(z'_P)\mathbf{R}_2(x'_P)\mathbf{R}_1(y'_P)\mathbf{R}_3(-\delta\psi)\mathbf{R}_3(-\psi_0)$$

$$\mathbf{R} = \mathbf{Q}_0(X,Y)\mathbf{R}_1(\delta Y' + y'_P)\mathbf{R}_2(-\delta X' + x'_P)\mathbf{R}_3(-\delta\psi + \delta Z' + z'_P)\mathbf{R}_3(-\psi_0)$$

Polar motion interpreted as precesion-nutation & LOD updates

### Miss-interpretation of precession-nutation

$$\mathbf{R} = \mathbf{Q}_0(X,Y)\delta\mathbf{R}(\mathbf{q}')\mathbf{R}_3(-\psi_0 - \delta\psi)\delta\mathbf{R}(\mathbf{p})$$

$\mathbf{q}'' = \mathbf{R}_3(\psi_0)\mathbf{q}' = [\delta Y'' \quad \delta X'' \quad \delta Z'']^T$

$$\mathbf{R} = \mathbf{Q}_0(X,Y)\mathbf{R}_3(-\psi_0 - \delta\psi)\delta\mathbf{R}(\mathbf{q}'')\delta\mathbf{R}(\mathbf{p})$$

$$\mathbf{R} = \mathbf{Q}_0(X,Y)\mathbf{R}_3(-\psi_0)\mathbf{R}_3(-\delta\psi)\mathbf{R}_3(\delta Z'')\mathbf{R}_2(-\delta X'')\mathbf{R}_1(\delta Y'')\mathbf{R}_2(x_P)\mathbf{R}_1(y_P)$$

$$\mathbf{R} = \mathbf{Q}_0(X,Y)\mathbf{R}_3(-\psi_0)\mathbf{R}_3(-\delta\psi + \delta Z'')\mathbf{R}_2(x_P - \delta X'')\mathbf{R}_1(y_P + \delta Y'')$$

Precession-nutation updates interpreted as polar motion & LOD update

### Miss-interpretation of some precession-nutation components

$$\mathbf{R} = \mathbf{Q}_0\mathbf{R}_2(\delta Y'_A + \delta Y'_B)\mathbf{R}_1(\delta X'_A + \delta X'_B)\mathbf{R}_3(-\psi_0 - \delta\psi)\mathbf{R}_2(x_P)\mathbf{R}_1(y_P) =$$

$$= \mathbf{Q}_0\mathbf{R}_2(\delta Y'_A)\mathbf{R}_1(\delta X'_A)\mathbf{R}_3(-\psi_0 - \delta\psi)\cdot \mathbf{R}_2(x_P - \sin\psi_0\delta X'_B + \cos\psi_0\delta Y'_B)\mathbf{R}_1(y_P + \cos\psi_0\delta X'_B + \sin\psi_0\delta Y'_B)$$

### Miss-interpretation of some polar motion components

$$\mathbf{R} = \mathbf{Q}_0\mathbf{R}_2(\delta Y')\mathbf{R}_1(\delta X')\mathbf{R}_3(-\psi_0 - \delta\psi)\mathbf{R}_2(x_{PA} + x_{PB})\mathbf{R}_1(y_{PA} + y_{PB}) =$$

$$= \mathbf{Q}_0\mathbf{R}_2(\delta Y' + \sin\psi_0 y_{PB} + \cos\psi_0 x_{PB})\mathbf{R}_1(\delta X' + \cos\psi_0 y_{PB} - \sin\psi_0 x_{PB})\cdot \mathbf{R}_3(-\psi_0 - \delta\psi)\mathbf{R}_2(x_{PA})\mathbf{R}_1(y_{PA})$$

**Conclusion:** Numerical separation of precession-nutation from polar motion within data analysis is **highly unstable** !

## COMPATIBLE SEPARATION OF EARTH ROTATION BY MATHEMATICAL MEANS

Mathematical separation of the rotation matrix **R** into precession-nutation, diurnal motion (LOD) and polar motion

$\vec{\omega}$ = rotation vector, with components $\boldsymbol{\omega}_C$ (celestial) and $\boldsymbol{\omega}_T$ (terrestrial)

$$[\boldsymbol{\omega}_C\times] = \frac{d\mathbf{R}}{dt}\mathbf{R}^T \qquad [\boldsymbol{\omega}_T\times] = \mathbf{R}^T\frac{d\mathbf{R}}{dt}$$

The mathematicaly induced **Compatible Celestial Pole** (**CCP**) has components

celestial

$$\mathbf{n}_C = \frac{1}{\omega}\boldsymbol{\omega}_C = \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \sqrt{1-\tilde{X}^2-\tilde{Y}^2} \end{bmatrix}$$

terrestrial

$$\mathbf{n}_T = \frac{1}{\omega}\boldsymbol{\omega}_T = \begin{bmatrix} \tilde{x}_P \\ -\tilde{y}_P \\ \sqrt{1-\tilde{x}_P^2-\tilde{y}_P^2} \end{bmatrix}$$

$$\omega = \sqrt{\boldsymbol{\omega}_C^T\boldsymbol{\omega}_C} = \sqrt{\boldsymbol{\omega}_T^T\boldsymbol{\omega}_T}$$

### COMPATIBLE EARTH ROTATION REPRESENTATION

$$\mathbf{R} = \mathbf{Q}_0(\tilde{X},\tilde{Y})\mathbf{R}_3(-\tilde{\psi})\mathbf{R}_2(\tilde{x}_P)\mathbf{R}_1(\tilde{y}_P)$$

where $\tilde{\psi} = \tilde{s}(\tilde{X},\tilde{Y}) - \tilde{\theta} - \tilde{s}'(\tilde{x}_P,\tilde{y}_P)$

$\tilde{\theta}(\tilde{\tau}) = A + B\cdot\widetilde{\mathrm{UT1}} = A + B(\mathrm{UTC} - \tilde{\tau})$

$\tilde{\tau} = \mathrm{UTC} - \widetilde{\mathrm{UT1}}$

COMPUTATIONS

$\mathbf{R}_3(-\tilde{\psi}) = \mathbf{Q}_0(\tilde{X},\tilde{Y})^{-1}\mathbf{R}\,\mathbf{R}_1(-\tilde{y}_P)\mathbf{R}_2(-\tilde{x}_P) \longrightarrow \tilde{\psi}$

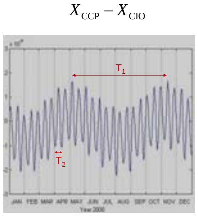
NRO conditions $\longrightarrow \tilde{s}(\tilde{X},\tilde{Y}) \quad \tilde{s}'(\tilde{x}_P,\tilde{y}_P)$

$\tilde{\theta} = \tilde{s}(\tilde{X},\tilde{Y}) - \tilde{s}'(\tilde{x}_P,\tilde{y}_P) - \tilde{\psi} \longrightarrow \tilde{\theta}$

$\tilde{\tau} = \mathrm{UTC} - \dfrac{\tilde{\theta}(\tilde{\tau}) - A}{B} \longrightarrow \tilde{\tau}$

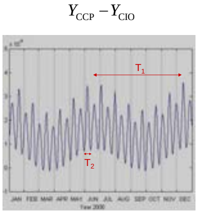
## COMPARISON OF THE COMPATIBLE CELESTIAL POLE (CCP) WITH THE CELESTIAL INTERMEDIATE POLE (CIP)

$X_{CCP} - X_{CIO}$



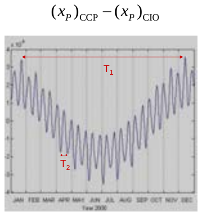

$T_1$ = 186.1 days $\quad T_2$ = 13.6 days

$Y_{CCP} - Y_{CIO}$



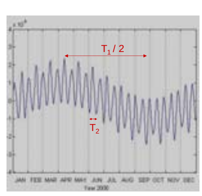

$T_1$ = 186.2 days $\quad T_2$ = 13.6 days

$(x_P)_{CCP} - (x_P)_{CIO}$




$T_1$ = 341.2 days $\quad T_2$ = 14.2 days

$(y_P)_{CCP} - (y_P)_{CIO}$




$T_1$ = 341.2 days $\quad T_2$ = 14.2 days

**Conclusions:**

Both celestial and terrestrial components of the CCP differ significantly from those of the CIP with 2 dominating terms having corresponding periods:

Precession-nutation

**186.2** days & **13.6** days

Polar motion

**341.2** days & **14.2** days

LOD is identical for CCP & CIP