The choice of reference system in ITRF formulation

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Abstract.. The problem of choosing an optimal reference system for the International Terrestrial Reference Frame (ITRF) is studied for both the rigorous solution which is a simultaneous stacking (removal of the reference system at each data epoch and implementation of a linear in time coordinate model) for all techniques, as well as for the usual numerically convenient separation into a set of individual stackings one for each technique and a final combination step for the derived initial coordinates and velocities. Two approaches are followed, an algebraic and a kinematic one. The algebraic approach implements the inner constraints, which minimize the sum of squares of the unknown parameters, as well as partial inner constraints, which minimize the sum of squares of a subset of the unknown parameters. In the kinematical approach the optimal minimal constraints are derived by requiring the minimization of the apparent coordinate variations: (a) with respect to the origin by imposing constant coordinates for the network barycenter, (b) with respect to orientation by imposing zero relative angular momentum for the network points conceived as mass points with equal mass and (c) with respect to the scale by imposing constant mean quadratic size (involving the distances of stations from their barycenter).

Key words: reference systems, ITRF, minimal constraints, inner constraints.

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1. Introduction

The implementation of an International Terrestrial Reference System (ITRS) by means of an International Terrestrial Reference Frame (ITRF) is based on the utilization of time series of station coordinates referring to different but overlapping subnetworks, one from each particular space technique (VLBI, SLR, GPS, DORIS). The object is to construct an optimal set of initial coordinates $\mathbf{x}_{0i}$ and velocities $\mathbf{v}_i$ for the stations P_i of the ITRF network which is the union of the subnetworks of all techniques. The adoption of the simple model of linear evolution in time

$$\mathbf{x}_i(t) = \mathbf{x}_{0i} + (t - t_0)\mathbf{v}_i \qquad (1)$$

(Altamimi et al., 2007, 2008) imposes a smooth temporal variation in order to remove noise from the input data, although systematic effects of various geophysical origins remain in the final ITRF residual series. With respect to the input coordinate data it is assumed that not only each technique refers to its own reference system but even each coordinate epoch refers to a separate reference system. The aim of this last rather strong assumption is the removal of systematic coordinate variations due to the temporal instability in the reference system definition. Thus the model for the observed coordinates $\mathbf{x}_{T,i}(t_k)$ of station P_i from technique T at epoch t_k has the form

$$\mathbf{x}_{T,i}(t_k) = \mathbf{f}_{T,i}(\mathbf{p}_{T,k}, \mathbf{x}_{0i}, \mathbf{v}_i) \qquad (2)$$

where in addition to the standard ITRF unknowns $\mathbf{x}_{0i}$ and $\mathbf{v}_i$, additional nuisance parameters appear, namely the transformation parameters $\mathbf{p}_{T,k}$ from the ITRF reference system to the one for each technique T and each coordinate input epoch t_k. In the most general case these involve 3 displacement components d_1, d_2, d_3, 3 rotation angles θ_1, θ_2, θ_3, and a scale factor $1+s$.

The determination of initial coordinates and velocities with simultaneous transformation of every epoch coordinates to a common reference system is usually referred to as "stacking" (Altamimi et a., 2007, 2008). Thus the ITRF formulation problem is in fact a simultaneous stacking for all the techniques, which involves a very large number parameters, most of which are the parameters $\mathbf{p}_{T,k}$, which are nuisance parameters in the ITRF formulation but they are needed for transforming earth orientation parameters from each epoch and technique to the reference system of the ITRF. In order to cope with the computational burden of a simultaneous stacking a two step approach is used instead (Altamimi et a., 2007, 2008). In the first step a stacking is performed for each technique T separately producing initial coordinates $\mathbf{x}_{T,0i}$ and velocities $\mathbf{v}_{T,i}$. In the second "combination" step the initial velocities from all techniques are combined to obtain the common ones of the ITRF. The model of the observations of the form

$$\mathbf{x}_{T,0i} = \mathbf{f}_{\mathbf{x}}(\mathbf{x}_{0i}, \mathbf{v}_i, \mathbf{q}_T), \qquad \mathbf{v}_{T,i} = \mathbf{f}_{\mathbf{v}}(\mathbf{x}_{0i}, \mathbf{v}_i, \mathbf{q}_T) \qquad (3)$$

involves the transformation parameters $\mathbf{q}_T$ from the ITRF reference system to that of the one of each technique T after its own stacking has been performed.

The observation models for the simultaneous stacking or the separate stackings and the combination have an inherent rank deficiency due to the lack of definition of the reference system. Indeed any change in the ITRF reference system by a particular transformation is counterbalanced by a change of the transformation parameters by the inverse transformation. If e.g. we write equation (2) in the form $\mathbf{x}_T = T(\mathbf{p})\mathbf{x}$ a coordinate transformation $\mathbf{x}' = T(\delta\mathbf{p})\mathbf{x}$ with inverse $\mathbf{x} = T^{-1}(\delta\mathbf{p})\mathbf{x}'$ leads to the model $\mathbf{x}_T = T(\mathbf{p}')\mathbf{x}'$ where the transformation parameters change from $\mathbf{p}$ to the ones $\mathbf{p}'$ implied by $T(\mathbf{p}') = T(\mathbf{p})T^{-1}(\delta\mathbf{p})$.

Therefore the rank deficiency must be overcome by choosing an optimal reference system among all possible ones. This is typically done in classical rigid geodetic networks by introducing additional constraints on the parameters which resolve the "choice of datum" problem. The fact that we are dealing with deformable networks requires the choice of an optimal reference system at each particular epoch among equivalent reference systems with coordinates connected by transformations $\mathbf{x}'(t) = T(\delta\mathbf{p}(t))\mathbf{x}(t)$ with parameters $\delta\mathbf{p}(t)$ which are smooth functions of time. A problem that arises in this respect is that general coordinate transformations $T(\delta\mathbf{p}(t))$ are not compatible with the linear time evolution model (1), since they transform coordinates $\mathbf{x}_i(t)$ into coordinates $\mathbf{x}'_i(t)$ which are not linear with respect to time t.

2. Observation equations for model-preserving and close to identity transformations

In order to overcome the above problems we follow the usual linearization procedure of replacing parameters with their corrections to known approximate values and neglecting of second and higher order terms. We also assume that the coordinate transformations involved are "close to the identity" so that only first order terms in the small coordinate transformation parameters $\mathbf{d} = [d_1\, d_2\, d_3]^T$, $\boldsymbol{\theta} = [\theta_1\, \theta_2\, \theta_3]^T$ and s are preserved. Since $\mathbf{R}(\boldsymbol{\theta}) \approx \mathbf{I} - [\boldsymbol{\theta}\times]$ a general coordinate transformation of the form $\mathbf{x}' = (1+s)\mathbf{R}(\boldsymbol{\theta})\mathbf{x} + \mathbf{d}$ becomes $\mathbf{x}' = \mathbf{x} + [\mathbf{x}\times]\boldsymbol{\theta} + s\mathbf{x} + \mathbf{d}$ or in terms of corrections $\delta\mathbf{x} = \mathbf{x} - \mathbf{x}^{\text{ap}}$ to approximate coordinates

$$\delta\mathbf{x}'(t) = \delta\mathbf{x}(t) + [\mathbf{x}^{\text{ap}}\times]\boldsymbol{\theta}(t) + s(t)\mathbf{x}^{\text{ap}} + \mathbf{d}(t)\,. \qquad (4)$$

The model (1) with $\mathbf{x}_i^{\text{ap}}(t) = \mathbf{x}_{0i}^{\text{ap}} + (t-t_0)\mathbf{v}_i^{\text{ap}}$ takes the form $\delta\mathbf{x}_i(t) = \delta\mathbf{x}_{0i} + (t-t_0)\delta\mathbf{v}_i$ and application of (4) to the general equation (2) yields the linearized observation equations for the stacking problem

$$\delta\mathbf{x}_i^k = \delta\mathbf{x}_{i0} + (t_k - t_0)\delta\mathbf{v}_i + s_k\mathbf{x}_{i0}^{\text{ap}} + [\mathbf{x}_{i0}^{\text{ap}}\times]\boldsymbol{\theta}_k + \mathbf{d}_k + \mathbf{e}_i^k \qquad (5)$$

where the observational noise $\mathbf{e}_i^k$ has also been taken into account while $\delta\mathbf{x}_i^k \equiv \mathbf{x}_{T,i}(t_k) - \mathbf{x}_{T,i}^{\text{ap}}(t_k)$. Dependence on the particular technique T has been dropped, while the subscript k denotes evaluation at epoch t_k.

The observation equations for the combination step are somewhat more involved. It can be shown (Altamimi & Dermanis, 2012) that only transformations preserving the linear model form (1) must be used which are the ones with parameters of the form

$$\mathbf{d}_T(t) = \mathbf{d}_{T0} + (t-t_0)\dot{\mathbf{d}}_T\,, \qquad (6a)$$

$$\boldsymbol{\theta}_T(t) = \boldsymbol{\theta}_{T0} + (t-t_0)\dot{\boldsymbol{\theta}}_T , \tag{6b}$$

$$s_T(t) = s_{T0} + (t-t_0)\dot{s}_T . \tag{6c}$$

With such linear in time parameter functions the observation equations for the combination step of the general form (3) become (Altamimi & Dermanis, 2012)

$$\delta\mathbf{x}_{T0i} = \delta\mathbf{x}_{0i} + [\mathbf{x}_{0i}^{\mathrm{ap}}\times]\boldsymbol{\theta}_{T0} + s_{T0}\mathbf{x}_{0i}^{\mathrm{ap}} + \mathbf{d}_{T0} + \mathbf{e}_{\mathbf{x}_{T0i}} , \tag{7a}$$

$$\delta\mathbf{v}_{Ti} = \delta\mathbf{v}_i + [\mathbf{x}_{0i}^{\mathrm{ap}}\times]\dot{\boldsymbol{\theta}}_T + \dot{s}_T\mathbf{x}_{0i}^{\mathrm{ap}} + \dot{\mathbf{d}}_T + \mathbf{e}_{\mathbf{v}_{Ti}} . \tag{7b}$$

where $\delta\mathbf{x}_{T0i} = \mathbf{x}_{T,0i} - \mathbf{x}_{0i}^{\mathrm{ap}}$ and $\delta\mathbf{v}_{Ti} = \mathbf{v}_{T,i} - \mathbf{v}_i^{\mathrm{ap}}$ are the reduced observations.

3. Constraints for the introduction of the optimal reference system

For the realization of the ITRF solution by either a simultaneous stacking with observation equations (5) or a two-step approach with stacking per technique using equations (5) followed by combination using equations (7), it remains to determine the minimal constraints which define the optimal reference system without affecting the optimal network shape at any epoch, which is uniquely defined by the least squares adjustment principle. There are two possible approaches: The first is a kinematic one where the optimality criterion is introduced directly by requiring that the variation of the coordinates is minimized in a specific way. The second is an algebraic one based on the inner constraints which minimize the sum of squares of all unknown parameters, or the partial inner constraints where the sum of selected parameters is involved. The kinematic constraints follow by requiring that the network barycenter remains constant and zero without loss of generality (definition of origin)

$$\mathbf{x}_B(t) \equiv \frac{1}{N}\sum_i \mathbf{x}_i(t) = \mathbf{0} , \tag{8a}$$

that the relative kinetic energy of the network stations (visualized as mass points of equal mass) is minimized or equivalently that the relative angular momentum is vanishing (definition of orientation)

$$\mathbf{h}_R(t) = \sum_i [\mathbf{x}_i(t)\times]\frac{d\mathbf{x}_i}{dt}(t) = \mathbf{0} \tag{8b}$$

and that the mean quadratic size S of the network defined by

$$S^2(t) = \sum_i \left[\mathbf{x}_i(t) - \mathbf{x}_B(t)\right]^T \left[\mathbf{x}_i(t) - \mathbf{x}_B(t)\right] \tag{8c}$$

remains constant (definition of scale).

The above optimality criteria lead to the following minimal constraints (Altamimi & Dermanis, 2012): For the definition of the system origin:

$$\frac{1}{N}\sum_i \delta\mathbf{x}_{0i} = -\overline{\mathbf{x}}_0^{\mathrm{ap}} \equiv -\frac{1}{N}\sum_i \mathbf{x}_{0i}^{\mathrm{ap}} , \tag{9a}$$

$$\frac{1}{N}\sum_i \delta\mathbf{v}_i = -\overline{\mathbf{v}}^{\mathrm{ap}} \equiv -\frac{1}{N}\sum_i \mathbf{v}_i^{\mathrm{ap}} \tag{9b}$$

For the definition of the system orientation:

$$\sum_i [\mathbf{x}_{0i}^{\mathrm{ap}}\times]\delta\mathbf{v}_i = -\mathbf{h}_R^{\mathrm{ap}} \equiv -\sum_i [\mathbf{x}_{0i}^{\mathrm{ap}}\times]\mathbf{v}_i^{\mathrm{ap}} \tag{10b}$$

For the definition of the system scale:

$$\sum_i (\mathbf{x}_{0i}^{ap} - \overline{\mathbf{x}}_0^{ap})^T \delta\mathbf{x}_{0i} = 0 \tag{11a}$$

$$\frac{1}{N}\sum_i (\mathbf{x}_{0i}^{ap} - \overline{\mathbf{x}}_0^{ap})^T \delta\mathbf{v}_i = (\overline{\mathbf{x}}_0^{ap})^T \overline{\mathbf{v}}^{ap} - \frac{1}{N}\sum_i (\mathbf{x}_{0i}^{ap})^T \mathbf{v}_i^{ap} \tag{11b}$$

Among the above constraints (9a) and (11a) define origin and scale, respectively, at the original epoch, while (9b), (10b) and (11b) define the rates of origin, orientation and scale, respectively. Note that the condition (8b) does not define the orientation at the initial epoch which must be also chosen by an additional constraint (to be borrowed from the next algebraic approach) in order to pick up a particular reference system orientation from an infinite number of dynamically equivalent ones satisfying (8b). Usually we choose $\mathbf{v}_i^{ap} = \mathbf{0}$ and if in addition approximate initial coordinates are chosen so that $\overline{\mathbf{x}}_0^{ap} = \mathbf{0}$ the constraints (9a), (9b), (10b), (11a), (11b) simplify, respectively, to

$$\sum_i \delta\mathbf{x}_{0i} = \mathbf{0} , \tag{12a}$$

$$\sum_i \delta \mathbf{v}_i = \mathbf{0} \tag{12b}$$

$$\sum_i [\mathbf{x}_{0i}^{\mathrm{ap}} \times] \delta \mathbf{v}_i = \mathbf{0} \tag{13b}$$

$$\sum_i (\mathbf{x}_{0i}^{ap})^T \delta \mathbf{x}_{0i} = 0 \tag{14a}$$

$$\sum_i (\mathbf{x}_{0i}^{ap})^T \delta \mathbf{v}_i = \mathbf{0} \tag{14b}$$

The algebraic approach follows the same general lines as in the case of rigid networks (Meissl, 1965, 1969, Blaha, 1971, Sillard and Boucher, 2001, Dermanis, 2003), with time independent coordinates. When observable quantities $\mathbf{y}$ are related to coordinate-related unknown parameters $\mathbf{x}$ by a linear(ized) model $\mathbf{y} = \mathbf{A}\mathbf{x}$, then the design matrix $\mathbf{A}$ has a rank deficiency equal to the number of coordinate transformation parameters which change $\mathbf{x}$ but leave $\mathbf{y}$ invariant (Grafarend and Schaffrin, 1976). A coordinate transformation with parameters $\mathbf{p}$ transforms the unknown $\mathbf{x}$ into new ones $\mathbf{x}' = T(\mathbf{p})\mathbf{x}$, which depend on both $\mathbf{x}$ and $\mathbf{p}$ through a linear(ized) relation of the form $\mathbf{x}' = \mathbf{x} + \mathbf{E}\mathbf{p}$. The derived matrix $\mathbf{E}$ determines the additional inner constraints $\mathbf{E}^T \mathbf{x} = \mathbf{0}$, which yield the unknown values satisfying $\mathbf{x}^T \mathbf{x} = \min$. Splitting the unknowns and the inner constraints in two sets $\mathbf{E}^T \mathbf{x} = \mathbf{E}_1^T \mathbf{x}_1 + \mathbf{E}_2^T \mathbf{x}_2 = \mathbf{0}$, we obtain the partial inner constraints $\mathbf{E}_1^T \mathbf{x}_1 = \mathbf{0}$, which satisfy $\mathbf{x}_1^T \mathbf{x}_1 = \min$.

In the stacking problem the unknowns are the initial coordinates and velocities $\mathbf{a}_i = [\mathbf{x}_{0i}^T \; \mathbf{v}_i^T]^T$ for each station P_i and the transformation parameters $\mathbf{z}_k = [\mathbf{d}_k^T \; \boldsymbol{\theta}_k^T \; s_k]^T$. A coordinate change with parameters $\mathbf{p} = [\mathbf{g}_0^T \; \boldsymbol{\psi}_0^T \; \lambda \; \dot{\mathbf{g}}^T \; \dot{\boldsymbol{\psi}}^T \; \dot{\lambda}]^T$, transforms the unknowns into

$$\mathbf{a}_i' = \mathbf{a}_i + \mathbf{E}_{\mathbf{a}_i} \mathbf{p} =$$

$$= \mathbf{a}_i + \begin{bmatrix} \mathbf{I} & [\mathbf{x}_{0i}^{\mathrm{ap}} \times] & \mathbf{x}_{0i}^{\mathrm{ap}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & [\mathbf{x}_{0i}^{\mathrm{ap}} \times] & \mathbf{x}_{0i}^{\mathrm{ap}} \end{bmatrix} \mathbf{p} \tag{15}$$

$$\mathbf{z}_k' = \mathbf{z}_k + \mathbf{E}_{\mathbf{z}_k} \mathbf{p} =$$

$$= \mathbf{z}_k + \begin{bmatrix} -\mathbf{I} & -(t_k - t_0)\mathbf{I} \end{bmatrix} \mathbf{p} \tag{16}$$

and the inner constraints $\sum_{i=1}^{N} \mathbf{E}_{\mathbf{a}_i}^T \mathbf{a}_i + \sum_{k=1}^{M} \mathbf{E}_{\mathbf{z}_k}^T \mathbf{z}_k = \mathbf{0}$ become

$$\sum_{i=1}^{N} \delta \mathbf{x}_{0i} - \sum_{k=1}^{M} \mathbf{d}_k = \mathbf{0} \tag{17a}$$

$$\sum_{i=1}^{N} \delta \mathbf{v}_i - \sum_{k=1}^{M} (t_k - t_0) \mathbf{d}_k = \mathbf{0} \tag{17b}$$

$$\sum_{i=1}^{N} [\mathbf{x}_{0i}^{\mathrm{ap}} \times] \delta \mathbf{x}_{0i} + \sum_{k=1}^{M} \boldsymbol{\theta}_k = \mathbf{0} \tag{18a}$$

$$\sum_{i=1}^{N} [\mathbf{x}_{0i}^{\mathrm{ap}} \times] \delta \mathbf{v}_i + \sum_{k=1}^{M} (t_k - t_0) \boldsymbol{\theta}_k = \mathbf{0} \tag{18b}$$

$$\sum_{i=1}^{N} (\mathbf{x}_{0i}^{\mathrm{ap}})^T \delta \mathbf{x}_{0i} - \sum_{k=1}^{M} s_k = 0 \tag{19a}$$

$$\sum_{i=1}^{N} (\mathbf{x}_{0i}^{\mathrm{ap}})^T \delta \mathbf{v}_i - \sum_{k=1}^{M} (t_k - t_0) s_k = 0\,. \tag{19b}$$

The partial inner constraints where only the parameters $\delta \mathbf{x}_{0i}$, $\delta \mathbf{v}_i = \mathbf{0}$ participate, are exactly the constraints (12a), (12b), (13b), (14a), (14b) plus the missing initial epoch orientation constraint (13a) which becomes

$$\sum_{i=1}^{N} [\mathbf{x}_{0i}^{\mathrm{ap}} \times] \delta \mathbf{x}_{0i} = \mathbf{0} \tag{13a}$$

In the combination problem the unknowns are again initial coordinates and velocities $\mathbf{a}_i = [\mathbf{x}_{0i}^T \; \mathbf{v}_i^T]^T$ as well as the transformation parameters $\mathbf{z}_T = [\mathbf{d}_{T0}^T \; \boldsymbol{\theta}_{T0}^T \; s_{T0} \; \dot{\mathbf{d}}_T^T \; \dot{\boldsymbol{\theta}}_T^T \; \dot{s}_T]^T$, which under a change of coordinate system transform according to

$$\mathbf{z}_T' = \mathbf{z}_T + \mathbf{E}_{\mathbf{z}_T} \mathbf{p} = \mathbf{z}_T - \mathbf{p} \tag{20}$$

and the inner constraints $\sum_i \mathbf{E}_{\mathbf{a}_i}^T \mathbf{a}_i + \sum_T \mathbf{E}_{\mathbf{z}_T}^T \mathbf{z}_T = \mathbf{0}$ with $\mathbf{E}_{\mathbf{a}_i}$ from (15) and $\mathbf{E}_{\mathbf{z}_T} = -\mathbf{I}$ become

$$\sum_{i=1}^{N} \delta \mathbf{x}_{0i} - \sum_{T=1}^{K} \mathbf{d}_{T0} = \mathbf{0} \tag{21a}$$

$$\sum_{i=1}^{N} \delta \mathbf{v}_i - \sum_{T=1}^{K} \dot{\mathbf{d}}_T = \mathbf{0} \tag{21b}$$

$$\sum_{i=1}^{N}[\mathbf{x}_{0i}^{\mathrm{ap}}\times]\delta\mathbf{x}_{0i}+\sum_{T=1}^{K}\boldsymbol{\theta}_{T0}=\mathbf{0} \tag{22a}$$

$$\sum_{i=1}^{N}[\mathbf{x}_{0i}^{\mathrm{ap}}\times]\delta\mathbf{v}_{i}+\sum_{T=1}^{K}\dot{\boldsymbol{\theta}}_{T}=\mathbf{0} \tag{22b}$$

$$\sum_{i=1}^{N}(\mathbf{x}_{0i}^{\mathrm{ap}})^{T}\delta\mathbf{x}_{0i}-\sum_{T=1}^{K}s_{T0}=0 \tag{23a}$$

$$\sum_{i=1}^{N}(\mathbf{x}_{0i}^{\mathrm{ap}})^{T}\delta\mathbf{v}_{i}-\sum_{T=1}^{K}\dot{s}_{T}=0\,. \tag{23b}$$

The partial inner constraints involving only initial coordinates and velocities are the same as in the stacking problem. The partial inner constraints involving only transformation parameters become

$$\sum_{T=1}^{K}\mathbf{d}_{T0}=\mathbf{0}\,,\qquad \sum_{T=1}^{K}\dot{\mathbf{d}}_{T}=\mathbf{0}\,, \tag{24}$$

$$\sum_{T=1}^{K}\boldsymbol{\theta}_{T0}=\mathbf{0}\,,\qquad \sum_{T=1}^{K}\dot{\boldsymbol{\theta}}_{T}=\mathbf{0} \tag{25}$$

$$\sum_{T=1}^{K}s_{T0}=0\,,\qquad \sum_{T=1}^{K}\dot{s}_{T}=0 \tag{26}$$

Of the above constraints (kinematic, inner, partial inner) only the ones related to the actual deficiencies of the reference system must be implemented. For example the origin and origin rate constraints do not apply to the SLR case where the geocenter is the known system origin. Since all techniques have their own scale, scale or scale rate constraints appear to be redundant. However since each technique has a different scale due to the different time unit realized through a different set of atomic clocks, these constraints should be incorporated into the combination step.

3. Conclusions

In comparison to inner or partial inner constraints the kinematic minimal constraints have the advantage of being independent of the approximate values of the parameters used in the linearization of the observation equations. They involve only station related parameters (initial coordinates and velocities), while inner constrains involve both station and reference system transformation parameters. Partial inner constraints may be formulated for either station or transformation parameters. The ones for station parameters may coincide with the kinematical ones (and thus share their independence from approximate values) if care is taken so that the used approximate values of velocities are zero and the approximate values of the initial coordinates have zero mean.

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