GEODENTIC PREDICTION OF CRUSTAL DEFORMATIONS AT THE SEISMIC AREA OF VOLVI

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Summary

Analysis techniques for plane crustal deformation, have been alternatively applied on the results of repeated triangulation surveys of an existing network at an active seismic zone in North Greece. Geodetic predictions of significant crustal deformation and comparisons of the results from several solutions have been as well tested.

1. INTRODUCTION

The application of geodetic methods for the extraction of crustal strain parameters is recognized as a useful technique in geophysical studies associated with earthquakes. There is a variety of methods treating this problem, each having its own particular advantages and disadvantages. For discussions on such problems and various applications see, e.g., Rikitake 1976, Bibby 1975, Brunner et al. 1981, Cohen and Cook 1979, Frank 1986, Livieratos 1981, Livieratos and Vlachos 1981, Prescott 1976, Reilly 1981, Savage 1978.

While the methods mentioned above obtain discrete results, this paper treats such problems using methods which starting from discrete values of displacement components, lead to continuous information about crustal strain parameters, i.e., maximum shear strain, dilatation, rotation, etc.

For an application of these methods the already available triangulation results from a network at the seismic zone of Volvi, Greece (Vlachos 1980), have been utilized.

The main intend of this paper is to elaborate the methods used, rather than to extract detailed geophysical conclusions. Short description of the methods is given in section 2. More details on the theoretical background and the development of the relevant formulae are given in Dermanis et al. 1981a. The examples given in section 3 have been selected as representative. For a more detailed study see Dermanis et al. 1981b.

** Topography, * Higher Geodesy and Cartography

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2. MATHEMATICAL TREATMENT

A short summary of the basic mathematical tools is given here for the derivation of crustal deformation parameters at any desired point of the area under consideration, utilizing discrete information, namely the displacement vector components \( (u_i, v_i) \) at the network stations. As an intermediate step the elements of the Jacobian of the displacements \( e_{xx} = \partial u / \partial x, \quad e_{xy} = \partial u / \partial y, \quad e_{yx} = \partial v / \partial x \)
and \( e_{yy} = \partial v / \partial y \), at any desired point are first derived and crustal deformation parameters, e.g., dilatation \( \Delta \), rotation \( \omega \), and maximum shear strain \( \gamma \), follow from the well known formulae

\[
\Delta = e_{xx} + e_{yy} \\
\omega = \left( e_{xy} - e_{yx} \right) / 2
\]
\[ Y = \sqrt{\left( e_{xx} - e_{yy} \right)^2 + \left( e_{xy} + e_{yx} \right)^2 / 4} \]

In the methods described here, a continuous field of displacement vectors \( u(x,y) \), \( v(x,y) \) is obtained through exact or smoothing interpolation techniques. The elements of the Jacobian of the displacements are obtained by simply taking the partial derivatives of the field.

In exact collocation the interpolated field is obtained with the help of a covariance kernel \( k(x,y;x',y') \) which is usually chosen to be isotropic. The relevant equations are

\[ u(x,y) = \sum_i k(x,y;x_i,y_i) \xi_i \]
\[ v(x,y) = \sum_i k(x,y;x_i,y_i) \eta_i \] (2)

where summation is taken over all stations of the network and \( \xi_i, \eta_i \) are components of vectors \( \xi, \eta \) obtained through the matrix equations

\[ \xi = K^{-1} u \]
\[ \eta = K^{-1} v \] (3)

\( K \) being the matrix with elements

\[ K_{ij} = k(x_i,y_i;x_j,y_j) \]

The elements of the Jacobian are generally obtained by formulae of the type

\[ \frac{\partial u}{\partial x}(x,y) = \sum_i \frac{\partial k}{\partial x}(x,y) \xi_i \] (4)

while in the case of an isotropic kernel \( k(r) \), where \( r = \sqrt{(x'-x)^2 + (y'-y)^2} \), (4) becomes

\[ \frac{\partial u}{\partial x}(x,y) = \sum_i \frac{x-x_i}{r_i} \frac{\partial k}{\partial r}(x,y) \xi_i \] (5)

with similar expressions for the other elements.
In smoothing collocation the above expressions are also applying, with the replacement of equation (3) with

\[
\begin{bmatrix}
\mathbf{g} \\
\eta
\end{bmatrix} = \begin{bmatrix}
K+C_{uu} & C_{uv} \\
C_{uv}^T & K+C_{vv}
\end{bmatrix}^{-1}
\begin{bmatrix}
u \\
v
\end{bmatrix}
\]

(6)

where \(C_{uu}, C_{vv}, C_{uv}\) are the covariance and cross-covariance matrices between the displacement components derived from the adjustments of the network at two epochs.

Analytical (minimum norm) interpolation is formulated as follows:
Find real coefficients \(a_s, b_s, s=1,2,...,m>2n\), where \(n\) is the number of the network stations, such that

\[
\begin{aligned}
\mathbf{u}(x,y) &= \sum_{s=1}^{m} a_s f_s(x,y) \\
\mathbf{v}(x,y) &= \sum_{s=1}^{m} b_s f_s(x,y)
\end{aligned}
\]

(7)

coincide with the given values \(u_i, v_i\) at the network points and the condition

\[
\sum_i (a_i^2 + b_i^2) = \min
\]

is satisfied; \(f_s(x,y), s=1,2,...,m\), is an a-priori selected family of functions. It can be proven that the solution to the above problem is given by the formulae of exact collocation if the covariance kernel

\[
k(x,y;x',y') = \sum_{s=1}^{m} f_s(x,y) f_s(x',y')
\]

is used. A smoothing version can be obtained exactly as in the case of exact collocation.

A method of smoothing interpolation is based on the use of moving weighted averages of the type

\[
\begin{aligned}
\mathbf{u}(x,y) &= \sum_{i=1}^{n} w(x,y;x_i^i,y_i^i) u_i \\
\mathbf{v}(x,y) &= \sum_{i=1}^{n} w(r_i) u_i
\end{aligned}
\]

(8)

with similar expression for \(v(x,y)\). In case of an isotropic weight function \(w(r)\) equation (8) becomes

\[
\begin{aligned}
\mathbf{u}(x,y) &= \sum_{i=1}^{n} w(r_i^i) u_i \\
\mathbf{v}(x,y) &= \sum_{i=1}^{n} w(r_i) u_i
\end{aligned}
\]

(9)
Elements of the Jacobian are computed by means of

$$\frac{\partial u}{\partial x} \bigg|_{(x,y)} = \frac{1}{W} \sum_{i=1}^{n} \frac{x-x_i}{r_i} \frac{\partial w}{\partial r} \bigg|_{r_i} (u_i - u(x,y))$$

(10)

and similar formulae, where

$$W = \sum_{i=1}^{n} w(r_i)$$

(11)

It is interesting to mention that equation (8), corresponds to a convolution with response function w(x,y;x',y') and input function a linear combination of Dirac impulses, i.e., a function with values u_i at network points and zero elsewhere.

Smoothing can also be applied a-posteriori on the results of the above methods using filtering techniques, for the rejection of undesired frequencies.

3. APPLICATION

An application of the above methods was carried out on the results of two successive triangulation of a sixteen point network established at the seismic zone of Volvi close to Thessaloniki (Fig. 1). The network was designed and insta-
lled just after the great earthquake of 1978; for relevant details see Vlachos 1980. The two campaigns considered included direction observations performed in July-August 1979 and repeated in April 1980. Note that an earthquake of magnitude M=4.6 occurred in March 1981 with epicenter at the central part of the network area (Fig. 1).

The observations for the two epochs were adjusted using the programme DEROS (Dermanis and Rossikopoulos 1981a), holding two stations fixed (minimal constraints). The location of the stations 3 and 4 (Fig. 1) to be held fixed was decided on geophysical grounds (Papazachos 1979).

An inner-constraints solution for the second epoch, using the adjusted coor-
dinates of the first epoch as approximate coordinates was obtained using the pro-
grame TRANSF (Dermanis and Rossikopoulos 1981b), which utilizes a generalized version of the Meissl-transformation (Meissl 1969). This approach allows the re-
moval from the coordinate differences between the two epochs of effects caused by a common translation, rotation and uniform scaling for all the network points
Fig. 2. Displacement vectors—Solution FIXED.

Fig. 3. Displacement vectors—Solution TRANS.
(similarity transformation). The relevant transformation parameters for the Volvi
network were:

Scale difference = $-4.35 \times 10^{-6}$

Rotation angle = 0°.86 = 4.2 mm/km

Translation in $x$ = 16.7 cm

Translation in $y$ = 6.0 cm.

Figs. 2 and 3 show respectively the displacement vectors as computed holding
two stations fixed (FIXED) and after the similarity transformation (TRANS).
In both cases the displacement were significant in comparison with the relevant a-
posteriori statistics.

The crustal parameter analysis here, is concentrated to the maximum shear
strain parameter, $\gamma$-patterns, due to its geophysical importance (see e.g.,
Livieratos 1980) and its frame invariance characteristics (Dermanis 1981).
For comparison reasons the results from the classical finite element method are
given in Fig. 4, using the programme CRUSTR (Livieratos and Tokmakidis 1981).

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Fig. 4. Maximum shear strain from the finite element method.

Values $\times 10^5$
Examining the results of collocation applied to the actual case (Figs. 5, 6, 7, 8, 9, 10), it is immediately seen that the \( \gamma \)-pattern for the FIXED and TRANS solutions remain practically unaltered, although the corresponding displacements are significantly different (Figs. 2, 3). This was expected since \( \gamma \) depends only on scale changes (Dermapis 1981) and as already mentioned the scale difference between the two solutions is very small (4 mm/km). Note that the \( \gamma \)-pattern values are an order of magnitude larger than the scale difference itself. In case distance measurements were included for both epochs there would be no scale difference (only rigid transformation allowed) and \( \gamma \)-patterns should coincide.

Another concern is the dependence of the results on the choice of the covariance kernel. In the results presented in Figs. 5 up to 12, exponential isotropic covariance kernel of the form

\[
k(r) = k(0) \exp(-\sigma^2 r^2) = \frac{k(0)}{2} \left(\frac{r}{R}\right)^2
\]

have been used. Three values of \( r \) (\( k(R)=1/2 \)), are used here, close to the mean distance, about 4 km, between neighbouring network stations.

There is no significant dependence of the results (Figs. 5 up to 10) on the chosen covariance kernel. There is an expected slight increase of smoothness in the central part of the network area with increasing \( R \), while edge effects are deteriorating. Dilatation and rotation patterns are also of interest (Figs. 11, 12). Results of analytical interpolation are very close to those of exact collocation for matching covariance kernels.

Concerning smoothing, the results depend strongly upon the ratio of the chosen \( k(0) \) value to the typical standard deviation of the displacement components (signal to noise ratio).

In the method of moving weighted averages, also isotropic exponential weight functions of type (12), have been used. A criterion for the choice of \( R \) is the agreement of the obtained residuals of the displacement components at the network points with their corresponding statistics. An example given in Fig. 13 shows similarity in pattern with the corresponding unsmoothed results, though magnitudes have decreased.

A-posteriori smoothing, i.e., rejection of high frequencies, is shown in Fig. 14, applied to the results of Fig. 5, using the programme LIZA (Livieratos and Zadro 1981). In Fig. 15 the previously omitted high frequencies are plotted.
Fig. 5. Maximum shear strain. Solution FIXED, R=4 km. Values x 10^5

Fig. 6. Maximum shear strain. Solution TRANS, R =4km. Values x 10^5
Fig. 7. Maximum shear strain. Solution FIXED, R=5 km. Values $\times 10^5$

Fig. 8. Maximum shear strain. Solution TRANS, R=5 km. Values $\times 10^5$
Fig. 9. Maximum shear strain. Solution FIXED, R=6 km. Values $\times 10^5$

Fig. 10. Maximum shear strain. Solution TRANS, R=6 km. Values $\times 10^5$
Fig. 11. Dilatation. Solution TRANS, R=4 km. Values x 10^5

Fig. 12. Rotation. Solution TRANS, R=4 km. Values x 10^5
Fig. 13. $\gamma$-pattern obtained from moving weighted averages, with exponential weight function. See eq. 12, with $R = 1.5$ km. Values $\times 10^5$.

Although this study is concentrated in analysis rather than on interpretation of physical features, the evident coincidence of maximum shear strain concentration with the epicenter of the above mentioned $M=4.6$ earthquake, allows to consider the $\gamma$-pattern as a probable premonitory event. The maximum $\gamma$ value of about $2 \times 10^{-5}$ for an eight month period corresponds to a $\gamma$ rate of the order of $3 \times 10^{-6}$ per year.

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Fig. 4. Maximum shear strain. Solution FIXED, R=4 km. Low pass filtering 8 km. Values x 10^5

Fig. 15. Maximum shear strain. Solution FIXED, R=4 km. High pass filtering 8 km. Values x 10^5
REFERENCES


Papazachos, B. C., 1979: Private Communication, Thessaloniki.


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