HISTORY OF CARTOGRAPHY, VOLUME 6

Geodetic Calculations: by Athanasios Dermanis

Geodesy is the science that studies the shape and gravity of the earth and their variation with time. Geodetic calculations process data from various types of observations in order to obtain optimal estimates of parameters describing the shape and gravity of the earth along with estimates of their accuracy. Coordinates of particular points are the parameters that describe the shape of the natural surface of the earth. However the term "shape of the earth" relates to the *geoid*, a fictitious surface remaining after an extension of the mean sea-level from the oceans to the continental part of the earth and the removal of the terrain relief. Since water remains in equilibrium when its free surface is everywhere perpendicular to the force of gravity, the determination of the shape of the earth as represented by the geoid is not a geometric problem but rather a problem of gravity field determination.

Knowledge of the gravity field is necessary for positioning using either classical or modern space techniques. Horizontal position (geodetic longitude and latitude) is determined by the direction perpendicular to a reference ellipsoid approximating the earth. Classical astronomical observations provide astronomical longitude and latitude, referring to the direction of the vertical. The deflection of the vertical from the ellipsoidal normal must be known in order to convert astronomical to the geodetic coordinates of cartographic practice. In 1928 F. A. Vening-Meinesz (1887-1966) extended the classical theory of G. G. Stokes (1819-1903) for geoid determination to the determination of deflections of the vertical using gravity observations. Height was determined independently by leveling techniques where consequent height differences were corrected for the effect of gravity and summed to determine height differences between permanent control points. Although data obtained from space techniques provide three-dimensional positioning, cartographic representation still requires the separation into horizontal position depicted on a map, and height represented by contour lines. In this respect heights above the ellipsoid provided by space techniques must be replaced by *orthometric* heights measured above the geoid, which is the proper zero-height reference surface. Thus presently gravity field determination, important in its own right, maintains its significance for positioning.

If we compare the beginning with the end of the 20th century, the situation with respect to the relation between theory and practice of geodetic calculations has been reversed. Presently the high accuracy and abundance of available observations poses significant challenges for both theoretical data handling techniques and appropriate mathematical modeling of relevant physical phenomena. One hundred years ago however geodesists had at their disposal a theoretical arsenal far beyond the observational and calculation capabilities of that time.

In the beginning of the century mapping was based on regional or national triangulation networks, where calculations were carried out with the help of logarithms, using the adjustment method of "condition equations" in order to limit the effect of observation errors. The obtained consistent adjusted values of observed angles and distances of a few baselines were used to compute coordinate estimates. Calculation difficulties necessitated many compromises, which did not allow the computation of the theoretically optimal solution, and even dictated simplified network designs consisting of triangle chains.

The first large scale effort for integrating regional networks into a unified datum was the North American Datum of 1927 (NAD27). The calculations for the adjustment of the western and eastern networks were completed in 1933. The next unification took place in Western Europe where observations of the RETrig network (1954-1979) were adjusted to obtain the European Datum of 1979 (ED79). The calculations involved 3,597 network points, 25,111 observations and 11,170 unknowns and achieved a relative accuracy of 1-2 meters. By that time advances in computers allowed the implementation of the method of “observation equations”, which allows the calculation of unknown coordinates directly from observations although some deviations from the theoretically optimal solution were still imposed by computational limitations. The replacement of invar wires by Electronic Distance Measurement (EDM) instruments, allowed a large number of baselines to be measured (2,732 or 11% of the total ED79 observations). The joint US-Canadian effort (1974-1986) led to the North American Datum of 1983 (NAD83), which covers the U.S., Mexico, Central America, Canada and Greenland. The U.S. network alone involved 259,000 points 1,734,000 observations and 929,000 unknowns. It was the last large scale geodetic effort before space techniques replaced the classical methods.

The introduction of EDM instruments was the last advancement in terrestrial methods. This technology started with the development of RADAR during World War II. In 1949, Dr. Erik Bergstrand of Sweden introduced the *geodimeter* (Geodetic Distance Measurement) that used light to measure distances up to 10km during daylight and 25km at night. In 1957 Dr. T. L. Wadley of South Africa introduced the *tellurometer* which used X-band radio waves to measure distances up to 50km. Distance measurement using lasers was also introduced in the mid 6Os. The relevant technology formed the basis for similar distance measuring techniques in space geodesy. EDMs were integrated with theodolites in the 80s into total stations appropriate for detailed surveying over small regions. However EDMs had practically no effect on the basic methods of geodetic calculations.

Despite practical difficulties geodetic calculation theory, driven by more modest surveying applications, witnessed some notable theoretical advances. One of them is related to the reliability of observations and in particular to the detection of blunders by the data snooping technique of W. Baarda (1917–2005). The use of planar coordinates for the analysis of observations capable of relative but not absolute positioning led to systems of equations with infinitely many solutions, one for every arbitrary definition of the coordinate system. A. Bjerhammer of Sweden introduced in 1951 the concept of generalized inverses of matrices already introduced by Moore in 1920, before their rediscovery and the consequent large development and application in modern mathematics by Penrose in 1955. Related is the work of P. Meissl (1934-1982) of the Graz Technical University, who clarified the relation between particular generalized inverse solutions and the use of additional constraints on the coordinates, in particular the inner constraints leading to the unique solution obtained by the unique generalized inverse called pseudoinverse in mathematical literature.

Another line of development related to gravimetric calculations of the height of the geoid above the reference ellipsoid. A series of theoretical developments took place in the 1950’s mainly at the Finnish Geodetic Institute or through their series of publications. A significant breakthrough is the work of T. Krarup (1919–2005) of the Danish Geodetic Institute who attacked the problem of interpolating gravity data with more advanced mathematical tools including the use Hilbert function spaces with reproducing kernels. This led to the possibility of processing simultaneously any gravity related observation in order to predict any desired gravity related quantity, using a technique which is called *collocation* within the geodetic community. The method became very popular thanks to its popularization by H. Moritz of the Graz Technical University and the software development by C. C. Tscherning of the Danish Geodetic Institute. Further elaborations of the probabilistic aspects of the method, in particular by F. Sanso of the Polytechnic of Milan, brought geodesy to the forefront of data analysis methods relating to unknown random fields, with similarities to the *kriging* method independently developed in geostatistics. Both approaches find their place within the general framework of prediction theory for stochastic processes independently pioneered in mathematics by N. Wiener (1894-1964) and A. N. Kolmogorov (1903-1987). In addition to the gravimetric problem, collocation applies to a wide variety of geodetic problems and is currently one of the most important tools for geodetic calculations.

The launch of the first artificial satellite of the earth on October 4, 1957, found the geodetic community ready to exploit the new possibilities. At the Department of Geodetic Sciences at the Ohio State University founded in 1951 by the Finnish geodesist W. Heiskanen (1895-1971), the research of G. Veis, W. Kaula (1926-2000) and I. Mueller developed the first computational techniques for the analysis of satellite tracking observations for both positioning and gravity field determination. Satellite positions serve as additional triangulation points, visible from widely separated stations, thus permitting the establishment of the first global geodetic networks with unprecedented accuracies. Starting from an accuracy of 20 meters a series of technological advances and data analysis techniques led to today's sub-centimeter positional accuracy. Analysis of satellite orbits driven by gravitational attraction led to the determination of the gravity field of the earth on a global scale. The first estimate related to the gravity field showed that the earth was less flat than previously believed.

After a short experimental period satellite geodesy became operational. The first period was dominated by satellite observations with ballistic cameras, where the satellite was photographed in the background of stars, providing the relative positions of worldwide tracking stations. This resulted in accuracies of the order 15-20 meters over the whole earth, with scale provided by terrestrial geodimeter distance observations. Soon other tracking techniques were introduced utilizing interferometry, electronic distance measurements and measurements based on the Doppler phenomenon, which allowed the determination of network scale and positioning with respect to the geocenter around which satellite orbits evolve.

The first technique to survive the test of time was laser tracking of satellites equipped with reflectors, now known as Satellite Laser Ranging (SLR), a method which was extended to the use of reflectors placed on the moon (Lunar Laser Ranging, LLR).. Laser ranging of satellites in low orbits brought a continuously improved knowledge of the earth's gravity field. This resulted in various "earth models", pioneered by R. Rapp at the Ohio State University, which are sets of gravity field parameters obtained from the combination of satellite and terrestrial data as well as satellite altimetry, a technique where observations of the distance between satellite and sea surface are used to determine the shape of the geoid over the oceans. In the 1970's the powerful method of Very Long Baseline Interferometry (VLBI) was developed which utilizes radio signals from extragalactic radio sources, to determine the shape of global networks and the rotation of the earth with a centimeter level accuracy. In 1990 the French introduced the DORIS system (Doppler Orbitography and Radiopositioning Integrated by Satellite) where satellites track a terrestrial network of beacons emitting radio signals utilizing the Doppler phaenomenon.

All the above techniques provided the basis for a unified high accuracy mapping of the earth, but they required instrumentation and research which was limited to specialized academic centers and governmental agencies. Of particular importance has been international cooperation, coordinated by the International Association of Geodesy (IAG) in collaboration with the Committee for Space Research (COSPAR). The obtained results had great scientific value but little effect on routine mapping activities. The situation was to change drastically when the first satellite of the NAVSTAR Global Positioning System (GPS) was launched in June 1977 marking the beginning of the GPS era for satellite geodesy. The ingenuity of geodetic researchers and instrumentation technologists must be praised for converting a system designed by the military for navigation with accuracy of 10-20 meters at best, into a geodetic system providing sub-centimeter accuracy. Such accuracy was achieved by exploiting observations on the carrier frequency rather than the digital codes used in navigation, a procedure which necessitates the determination of the number of unknown integer wavelengths contained in the satellite-to-receiver distance (integer ambiguity).

In 1994, the IAG established the International GPS Service (IGS), which utilizes data from an extensive worldwide network of about 350 permanent stations and various data analysis centers, to provide high accuracy orbit and atmospheric condition information for use in professional mapping. Many countries are establishing additional densification of GPS networks, obtaining high accuracy by using a single receiver, instead of two, in combination with data from a nearby permanent station. The evolution of satellite geodesy calculations started with great computational difficulties but was eventually boosted by the exponential growth of computer capabilities. Today GPS positioning calculations are carried out by relatively inexpensive commercial software using modest personal computers. On the other hand auxiliary data provided by the scientific community are the result of long elaborate modeling and numerical procedures. Although computational cost is no more of concern, the difficulties lie in the organization, handling, and evaluating an ever increasing huge amount of data and the development of efficient physical mathematical and statistical models. The basis of highly accurate global positioning is the International Terrestrial Reference Frame (ITRF) consisting of the time variable coordinates of a very large number of fundamental stations involved in various space techniques (VLBI, SLR, DORIS, and GPS). Coordinates at a reference epoch, constant station velocities and earth rotation parameters from the particular techniques are optimally combined at the International Earth Rotation and Reference System Service (IERS), a common service of the IAG and the International Astronomical Union (IAU).

Before the end of the 20th century GPS was already dominating professional applications. The traditional triangulation, trilateration and traverse techniques based on theodolites and EDM instruments were gradually abandoned. GPS provides high accuracy positioning by slow static methods (10-20 minutes per point) with elaborate post-processing calculations implementing auxiliary data provided by the IGS. Less accurate results can be achieved with greater speed in static or kinematic mode where the receiver is moving aboard a vehicle. Calculations must be done in real time in order to ensure that the integer ambiguity has been resolved before proceeding any further. Real time operation requires data transmission through mobile phone connections between receivers or with a permanent station when a single receiver is used. The problem of data analysis where the unknown parameters include integer numbers has been the subject of much theoretical research aiming at producing very fast algorithms. The most successful results have been produced by the group of P. Teunissen at the Technical University of Delft.

The beginning of the 21st century finds professional mapping practice revolutionized with the use of GPS and its Russian counterpart GLONASS with even higher expectations from the newly planned European GALILEO system. Lack of satellite visibility in urban areas is partly resolved by the much promising *pseudolites* (pseudo-satellites), which are ground based transmitters of signals similar to those of satellites.

High accuracy position determination has found applications other than those of traditional less demanding mapping. Mostly these are of geophysical interest, such as the monitoring of crustal deformations for hazard prevention with simultaneous contributions from high accuracy determination of the gravity field. The geodetic community expects a lot from the GOCE mission of ESA, in operation since 2009, where the change-rate of gravity vector components will be measured by a gradiometer aboard a low height satellite. Already valuable data have been successfully analyzed from NASA's GRACE mission which uses satellite-to-satellite tracking between twin satellites, which are also equipped with GPS receivers.

To meet challenges much more demanding than mapping (deformation of the solid Earth, mass transport in the Earth system, Atmosphere-Ocean dynamics, Global water cycle), the IAG established a special project the Global Geodetic Observing System (GGOS), as a part of UNESCO's Integrated Global Observing Strategy Partnership (IGOS-P). The GGOS project poses great challenges for innovative geodetic calculation techniques where a precise modeling of complicated geophysical phenomena is required. The great successes of space geodesy observation and data analysis techniques within the last four decades of the 20th century, provides the basis for great hopes in meeting these new challenges of the 21st century.

Bibliography

Baarda, Willem. 1968. *A testing procedure for use in geodetic networks*. Netherlands Geodetic Commission. Publications on Geodesy 9, Vol. 2 Nr. 5. Delft.

Bjerhammar, A. 1951. "Rectangular reciprocal matrices with special references to geodetic calculations". *Bulletin Geodesique* 52:188-220.

Bomford, Guy. 1962. *Geodesy*. 2d ed. Oxford: Oxford University Press.

Grafarend, Erik and Burkhardt Schaffrin. 1993. *Ausgleichungs-rechnung in linearen Modellen*. Mannheim: Bibliographisches Institut Wissenschaftsverlag.

Heiskanen, W. A. and H. Moritz. 1967. *Physical Geodesy*. San Francisco: W.H. Freeman and Company.

Hofmann-Wellenhof, B., Lichtenegger, H. and Collins, J. 2001. *Global Positioning System. Theory and Practice*. 5th ed. Rev., Wien-New York: Springer.

Karup, Torben. 1969. A *Contribution to the Mathematical Foundation of Physical Geodesy*. Copenhagen: Geodaetisk Instituts, Med. No. 44.

Leick, Alfred. 1994. *GPS Satellite Surveying.* 2d ed. New York: John Wiley & Sons, Inc.

Moritz, Helmut. 1980. *Advanced Physical Geodesy*. Karlsruhe: Wichmann Verlag.

Mueller, Ivan. [1996]: *Commission on International Coordination of Space Techniques for Geodesy and Geodynamics (CSTG). A Historical Note*. International Association of Geodesy.

Sansò, Fernando. 1978. "The Minimum Mean Square Estimation Error Principle in Physical Geodesy. Stochastic and Non-Stochastic Interpretation". In *Proceedings 7th Symposium on Mathematical Geodesy*. Firenze: Istituto Geografico Militare.

Teunissen P.J.G. 2004. "Integer Least-Squares". In *V Hotine-Marussi Symposium on Mathematical Geodesy*, ed. F. Sansò ", International Association of Geodesy Symposia, 127: 69-80. Berlin-Heidelberg-New York: Springer-Verlag,.

Torge, Wolfgang. 2001. *Geodesy*. 3rd ed, rev. & extend. Berlin-New York: deGruyter